

Exercise 70

For the following exercises, solve the absolute value equation.

$$\left| \frac{1}{3}x + 5 \right| = \left| \frac{3}{4}x - 2 \right|$$

Solution

Start by removing the absolute value sign on the right. Do so by placing \pm on the left side.

$$\pm \left| \frac{1}{3}x + 5 \right| = \frac{3}{4}x - 2$$

$$\left| \frac{1}{3}x + 5 \right| = \frac{3}{4}x - 2 \quad \text{or} \quad - \left| \frac{1}{3}x + 5 \right| = \frac{3}{4}x - 2$$

Multiply both sides of the second equation by -1 to isolate the absolute value sign.

$$\left| \frac{1}{3}x + 5 \right| = \frac{3}{4}x - 2 \quad \text{or} \quad \left| \frac{1}{3}x + 5 \right| = -\frac{3}{4}x + 2$$

Remove the absolute value signs by placing \pm on the right side of each equation.

$$\frac{1}{3}x + 5 = \pm \left(\frac{3}{4}x - 2 \right) \quad \text{or} \quad \frac{1}{3}x + 5 = \pm \left(-\frac{3}{4}x + 2 \right)$$

$$\frac{1}{3}x + 5 = \left(\frac{3}{4}x - 2 \right) \quad \text{or} \quad \frac{1}{3}x + 5 = - \left(\frac{3}{4}x - 2 \right)$$

$$\text{or} \quad \frac{1}{3}x + 5 = \left(-\frac{3}{4}x + 2 \right) \quad \text{or} \quad \frac{1}{3}x + 5 = - \left(-\frac{3}{4}x + 2 \right)$$

Distribute the minus signs.

$$\frac{1}{3}x + 5 = \frac{3}{4}x - 2 \quad \text{or} \quad \frac{1}{3}x + 5 = -\frac{3}{4}x + 2 \quad \text{or} \quad \frac{1}{3}x + 5 = -\frac{3}{4}x + 2 \quad \text{or} \quad \frac{1}{3}x + 5 = \frac{3}{4}x - 2$$

Notice that the first and third equations are the same and that the second and fourth equations are the same.

$$\frac{1}{3}x + 5 = \frac{3}{4}x - 2 \quad \text{or} \quad \frac{1}{3}x + 5 = -\frac{3}{4}x + 2$$

Solve for x in each equation.

$$\frac{1}{3}x + 5 = \frac{3}{4}x - 2 \quad \text{or} \quad \frac{1}{3}x + 5 = -\frac{3}{4}x + 2$$

$$5 + 2 = \frac{3}{4}x - \frac{1}{3}x \quad \text{or} \quad 5 - 2 = -\frac{3}{4}x - \frac{1}{3}x$$

$$7 = \frac{5}{12}x \quad \text{or} \quad 3 = -\frac{13}{12}x$$

$$\frac{84}{5} = x \quad \text{or} \quad -\frac{36}{13} = x$$

Therefore,

$$x = \left\{ -\frac{36}{13}, \frac{84}{5} \right\}.$$